Collision Resolution in Slotted ALOHA with Multi-User Physical-Layer Network Coding

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***Abstract*—Two new schemes are proposed for collision resolution in slotted ALOHA networks based on multi-user physical-layer network coding (MU PHY NC). In the proposed random access schemes, a collision of a generic number of packets can be recovered decoding the XOR of the original messages, such that the signal resulting from the collision is exploited rather than being discarded. Two different schemes that differ in terms of the amount of control information that needs to be transmitted from the access point, are studied.**

***Index Terms*—Physical-layer Network Coding, slotted ALOHA, multiple access, collision resolution, satellite communications.**

I. Introduction

Multiple access systems, and particularly ALOHA-like systems, constitute an essential component of practical wireless communication systems. Popular examples are the multiple access to an access point in wireless local area networks (WLANs), access to a base station in a cellular system and multiple access to a satellite. It is well known that the throughput of a slotted ALOHA system is limited to 37% of that of a centralized system due to collisions of signals transmitted by two or more nodes accessing the channel simultaneously. The possibility of recovering packets involved in a collision is addressed in [1], where a collision resolution technique based on interference cancelation is proposed, which is extended in [2]. In [3] interference cancellation at the receiver is performed by exploiting the information on the phase shift and channel attenuation of each of the colliding signals. Another technique used for collision recovery is physical layer network coding (PHY NC) [4], [5], that has been applied to the two-way relay channel (TWRC) as well as the M-way relay channel [6], [7].

We propose two new schemes for collision resolution in slotted ALOHA networks, in which PHY NC is applied to recover collisions at the receiver. We call this approach multi-user physical-layer network coding (MU PHY NC).

The rest of the paper is organized as follows: in Section II we introduce the system model. In Section III we describe two collision recovery protocols based on MU PHY NC. In Section IV we address some issues regarding the practical implementation of the proposed protocols. Section V contains the numerical results, while Section VI concludes the paper.

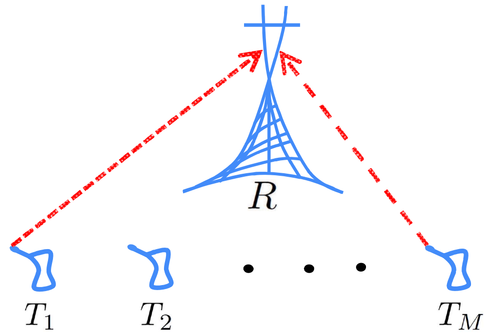

Fig. 1. Multiple access system with M transmitters and one receiver.

II. System Model

Let us consider a system with M transmitters $T_1, ..., T_M$ and one receiver R as the one depicted in Fig. 1. Packet arrivals at each transmitter is modeled as a Poisson process with rate $\frac{\lambda}{M}$ independent from the other transmitters. Each packet $\mathbf{u}_i = [u_i(1), ..., u_i(K)]$ consists of K binary symbols of information $u_i(j) \in \{0, 1\}$ for $j = 1, \dots, K$. We assume that, upon receiving a message, each terminal T_i uses the same linear channel code of fixed rate $r = \frac{K}{N}$ to protect its message $\mathbf{u}_i$ obtaining the codeword $\mathbf{x}_i = [x_i(1), ..., x_i(N)]$, where $x_i(t) \in \{0, 1\}$ for $t = 1, \dots, N$. For ease of exposition BPSK modulation is considered. Each codeword $\mathbf{x}_i$ is BPSK modulated (using the mapping $0 \to -1$, $1 \to +1$), thus, obtaining the transmitted signal vector $\mathbf{s}_i = [s_i(1), ..., s_i(N)]$ with $s_i(t) \in \{-1, +1\}$ for $t = 1, \dots, N$.

When more than one transmitter has a packet to transmit, a collision occurs at the receiver. A collision involving k transmitters is said to have size k. We consider a block fading channel model in which the channel from each transmitter to the receiver has a Rayleigh distribution, independent from other channels, and its value remains constant for a block of N channel uses, and changes independently from one channel block to the next one. The received signal at receiver R in case of a collision of size k is (assuming, without loss of generality, the first k terminals collide) is given by

$$\mathbf{y} = h_1\mathbf{s}_1 + h_2\mathbf{s}_2 + ... + h_k\mathbf{s}_k + \mathbf{w} = \mathbf{h}^T\mathbf{S} + \mathbf{w}, \qquad (1)$$

where $\mathbf{h}^T = [h_1, \dots, h_k]$ is the vector of channel coefficients, which are independent and identically distributed (i.i.d.) with circularly symmetric complex Gaussian distribution accounting for channel fading and path loss, and $\mathbf{S}$ is a matrix obtained stacking up signal vectors $\mathbf{s}_1 \ldots \mathbf{s}_k$, while $\mathbf{w}$ is an additive white Gaussian noise (AWGN) process with variance σ^2. In case of a collision, signals from the transmitters add up with symbol synchronism. This can be achieved through orthogonal frequency division multiplexing (OFDM) modulation, that can help to counteract the delay spread in signal propagation. We further assume that the receiver has the knowledge of the nodes that are transmitting as well as the full channel state information at each time slot. This can be achieved using a CDMA-encoded preamble in each transmitted signal, assuming that the probability that two nodes use the same code is negligible [1], and adding in the preamble of the first time slot the seed for the random number generator used by each node to determine its transmission sequence.

When a collision occurs at the receiver, it tries to decode the bit-wise XOR of the transmitted messages. We call this approach multi-user PHY NC (MU PHY NC). This can be done by feeding the decoder with the log-likelihood ratios (LLR) for the received signal. Such LLRs can be calculated as follows in case LDPC codes and BPSK modulation are used (see [8] and [9] for an extension to higher order modulations). When signals from k transmitters collide, the received signal at R is given by (1). Each codeword $\mathbf{x}_i$ is calculated from $\mathbf{u}_i$ as $\mathbf{x}_i^T = \mathbf{u}_i^T \mathbf{G}$, where $\mathbf{G}$ is the K by N generator matrix of the common code. All nodes use the same matrix $\mathbf{G}$. Starting from $\mathbf{y}$ the receiver R wants to decode the codeword $\mathbf{x}_s \triangleq \mathbf{x}_1 \oplus \mathbf{x}_2 \oplus \ldots \oplus \mathbf{x}_k$, where $\oplus$ denotes the bit-wise XOR. In order to do this the LDPC decoder of R is fed with the vector $\mathbf{L}^{\oplus} = [L^{\oplus}(1), ..., L^{\oplus}(N)]$ of LLRs for $\mathbf{x}_s$. After some calculations we find the following expression for the LLR:

$$L^{\oplus}(t) = \ln \left\{ \frac{\sum_{i=1}^{\lfloor \frac{k+1}{2} \rfloor} \sum_{m=1}^{\binom{k}{2i-1}} e^{-\frac{\left| y(t) - \mathbf{d}^{o}(2i-1,m)^T \mathbf{h} \right|^2}{2\sigma^2}}}{\sum_{i=1}^{\lfloor \frac{k+1}{2} \rfloor} \sum_{m=1}^{\binom{k}{2i}} e^{-\frac{\left| y(t) - \mathbf{d}^{e}(2i,m)^T \mathbf{h} \right|^2}{2\sigma^2}}} \right\}, \quad (2)$$

where $y(t)$ is the t-th element of vector $\mathbf{y}$ while $\mathbf{d}^{o}(2i-1, m)$ and $\mathbf{d}^{e}(2i, m)$ are column vectors containing one (the m-th) of the $\binom{k}{2i-1}$ or $\binom{k}{2i}$ possible permutations over k symbols (without repetitions) of an odd or even number of symbols with value "+1", respectively. Equation (2) is derived considering that an even or an odd number of symbols with value $+1$ adding up at R must be interpreted by the decoder as a 0 or a 1, respectively. If the decoding process is successful, R obtains the message $\mathbf{u}_s \triangleq \mathbf{u}_1 \oplus \ldots \oplus \mathbf{u}_k$. In Fig. 2 the frame error rate (FER) values for different numbers of transmitters obtained using these LLR values are depicted. The plots are obtained using a non-systematic LDPC code with rate $1/2$ and codeword length equal to 480 symbols. It can be seen from Fig. 2 how the FER slightly grows with the number of users, such that the loss gets smaller as the number of users grows. As a matter of facts, fixing a FER of 10^{-2}, there is a loss of about 1.5 dB when going from 1 to 2 transmitters and just 0.3 dB going from 5 to 20 transmitters.

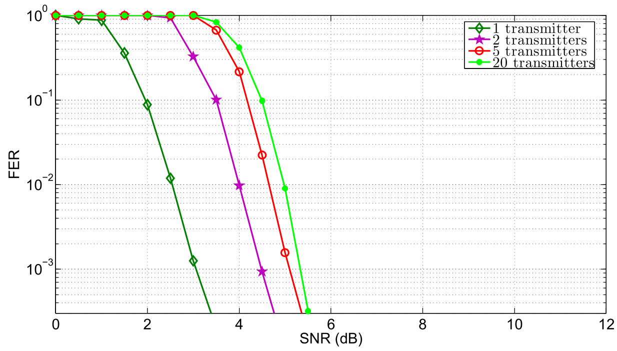

Fig. 2. FER for decoding the XOR using LLRs for different number of transmitters when the channel gains are equal. The SNR in the plot is that of a single user transmission. A non-systematic LDPC code with rate $1/2$ and codeword length 480 symbols is used.

III. Multi-user Physical-layer Network Coding Protocols

Now we introduce two protocols for collision recovery for the fading multiple access system introduced in Section I.

A. Terrestrial MU PHY NC

In terrestrial MU PHY NC, the receiver R broadcasts a *reservation message* (RM) when a collision occurs, thus starting a *recovery phase* (RP). During the RP, each node involved in the collision accesses the channel in each time slot with probability q, while nodes not involved in the collision stop transmitting. The RP goes on until R decodes k linearly independent messages, which allows it to recover all of the colliding messages. At this point a block ACK is sent out by R, signifying the end of the RP and the beginning of a new access phase. In order to clarify how the proposed scheme works, let us consider the following example.

Example: let us consider a network with $M = 4$ nodes. In slot number n nodes T_1, T_2 and T_3 access the channel simultaneously, thus leading to a collision of size $k = 3$. R broadcasts an RM starting an RP. The RM will prevent node T_4 from transmitting until the end of the RP. During the RP the nodes involved in the collision access the channel with probability q. Let us assume that the random access results in the access pattern shown in Table I, where the 1s indicate nodes accessing the channel in a given slot while 0s indicate nodes that remain silent. With reference to the table, in time slot n the collision takes place with three nodes accessing the channel. If decoding is successful, R obtains $\mathbf{u}_s[n] = \mathbf{u}_1 \oplus \mathbf{u}_2 \oplus \mathbf{u}_3$. In slot $n+1$ only nodes T_1 and T_3 access the channel, and R obtains $\mathbf{u}_s[n+1] = \mathbf{u}_1 \oplus \mathbf{u}_3$. In slot $n+2$ just node T_2 transmits. In this case the message $\mathbf{u}_s[n+2] = \mathbf{u}_2$ obtained by R is not linearly independent from the two previously received vectors, as it can be obtained as $\mathbf{u}_s[n+2] = \mathbf{u}_s[n] \oplus \mathbf{u}_s[n+1]$. Thus, R does not stop the RP, and waits for another transmission. In slot $n+3$ nodes T_1 and T_2 access the channel and R obtains $\mathbf{u}_s[n+3] = \mathbf{u}_1 \oplus \mathbf{u}_2$. At this point R is able to recover all of the original messages. It can, for instance, XOR $\mathbf{u}_s[n+2]$ and $\mathbf{u}_s[n+3]$ to obtain

TABLE I

EXAMPLE OF ACCESS PATTERNS FOR COLLISION RECOVERY IN CASE OF A COLLISION OF SIZE $k = 3$. THE 1S INDICATE WHEN THE NODES ACCESS THE CHANNEL WHILE 0S INDICATE WHEN THE NODES REMAIN SILENT.

	T_1	T_2	T_3
Slot n	1	1	1
Slot n + 1	1	0	1
Slot n + 2	0	1	0
Slot n + 3	1	1	0

$\mathbf{u}_1$, then XOR $\mathbf{u}_s[n]$ and $\mathbf{u}_s[n+3]$ to obtain $\mathbf{u}_3$, while $\mathbf{u}_2$ is already decoded at the end of slot $n+2$. In general, the receiver will be able to obtain the original messages when the matrix obtained piling up the encoding vectors (rows in Table I) has rank k. The encoding vectors are known to R under the hypothesis that the receiver knows which terminals collide.

B. Satellite MU PHY NC

In satellite MU PHY NC, when a collision occurs, the receiver calculates the number N_{sl}^{opt} of transmission slots needed for receiving k independent packets that maximizes the throughput:

$$N_{sl}^{opt} = \arg\max_{N_{sl}} \left[k \frac{1 - p_{bl}(N_{sl}, FER)}{N_{sl}} \right], \quad (3)$$

where $p_{bl}(N_{sl}, FER)$ is the probability of not decoding a given block in N_{sl} slots for a given FER. The FER can be assumed to be zero if the SNR is sufficiently high. The value N_{sl}^{opt} is then broadcasted to all of the users, and an RP starts. Again, only nodes involved in the collision access the channel in each of the N_{sl}^{opt} time slots with probability q. If R receives k innovative packets in N_{sl}^{opt} transmissions it is able to decode all of the original messages, otherwise the whole block is lost. At this point a new random access phase starts. This scheme has the advantage that only one message from the satellite is sufficient for coordinating the recovery, which saves 250 milliseconds of round trip time (RTT) in case of communications with a GEO satellite with respect to the terrestrial scheme [10].

IV. IMPLEMENTATION ASPECTS

Hardware limitations may bound the maximum size of collisions that can be recovered in a real system. One possible limiting factor is the dynamic range of the receiver front end. If the power of the received signal is too high, the distortion introduced by the front end on the received signal may lead to an unacceptably high FER. Let us denote by $\overline{P}$ the maximum input power for which the distortion is "reasonably" low, i.e., the increase in FER does not exceed a target value, and by P_s the power at the receiver's input when only one node transmits. Assume, for simplicity, that the path loss from each node to the receiver is almost the same (for instance this is the case for GEO satellite networks). Assuming independent transmissions across nodes, an approximate estimation of the maximum recoverable collision size is $\overline{k} \triangleq \frac{\overline{P}}{P_s}$. Another limiting factor is the channel estimation algorithm, the performance of which may degrade with k.

Overall, it is reasonable that in a real system the recoverable collision size is bounded by a certain $\overline{k}$, such that the collisions of size greater than $\overline{k}$ can be recovered with low probability. Note that this limitation is not present if the number of terminals in the networks is less $\overline{k}$. In systems with a larger number of terminals the proposed schemes can recover collisions of lower size. As the average size of the collisions grows with the offered load, we expect a drop in throughput at high loads. Numerical results in Section V confirm this intuition. The exact value of $\overline{k}$ depends greatly on the considered system, as it may be affected by several factors such as the disposition of the terminals around the receiver, distribution of the fading process, accuracy of the channel estimation algorithm and hardware characteristics of the transmitters and the receiver.

An additional remark must be made for the terrestrial MU PHY NC scheme. We stated in Section II that an RP starts when the satellite broadcasts an RM. However, a time equal to a whole RTT passes between the moment the first colliding packets are transmitted and the RM is received by the terminals. Since the RTT for a satellite in geosynchronous orbit is about 250 msec, several packets will be probably transmitted during this time, possibly originating other collisions. In order to overcome this problem, a terminal must keep track of packets transmitted up to approximately $\psi = |RTT| \times \overline{k}$ slots before, where $|RTT|$ represents the number of time slots in an RTT. The value of ψ is obtained by considering the worst case scenario, i.e. a situation in which for $|RTT|$ consecutive slots a collisions of size exactly $\overline{k}$ occurs, and that, about $\overline{k}$ retransmissions are needed to recover each of them.

Note that, although we consider a time division multiple access (TDMA) scheme, the proposed techniques can be also applied to other access schemes, such as multi-frequency-TDMA (MF-TDMA) or code division multiple access (CDMA), in which the proposed techniques can be used to recover collisions in each of the frequency/code sub-channels.

V. NUMERICAL RESULTS

In this section we describe the simulation setup used to compare the performance of the proposed schemes through numerical analysis. We assume that during the recovery phase new packets may arrive at the transmitters, but can not be transmitted until the recovery phase ends. We further assume that each transmitter can buffer a maximum of B packets, after which new packets that arrive are dropped. Our performance metric is the system throughput Φ defined as:

$$\Phi = \lambda(1 - \Upsilon), \quad (4)$$

where Υ is the average packet loss rate (i.e. the ratio of lost packets to the total number of packets that arrive at the transmitters) due to buffer overflow at the transmitters and incorrect decoding at R. As a benchmark we consider a slotted ALOHA system with Poisson arrivals and with neither collision recovery nor backlogging.

In Fig. 3, Φ is plotted against λ. The proposed schemes achieve a higher throughput with respect to the ALOHA

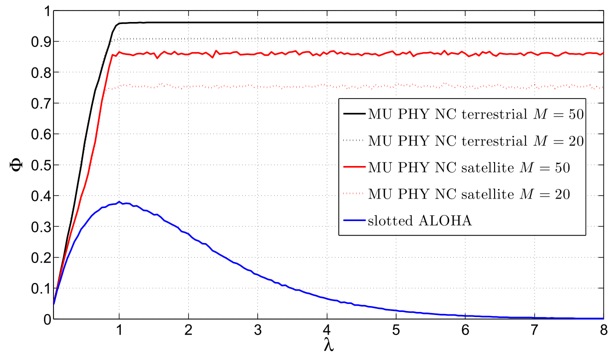

Fig. 3. System throughput with respect to packet arrival rate λ with $B = 10$ and $q = \frac{1}{2}$. The terrestrial system achieves a throughput higher than that of the satellite system due to its lower packet loss rate.

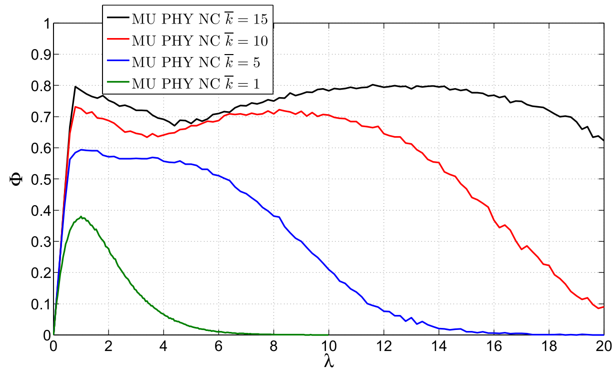

Fig. 4. System throughput with respect to packet arrival rate λ in case of maximum recoverable collision size equal to $\overline{k}$. A terrestrial system with $M = 10^5$ users, $B = 10$ and $q = \frac{1}{2}$ is considered.

system and keep a steady throughput at high packet arrival rates. The value of the steady throughput increases with the total number of nodes in the network. This is because the relative number of extra transmissions needed to get k innovative packets decreases with k. As a matter of fact, it can be proved that the probability of recovering k innovative packets with $n = k + e, e > 0$ transmissions is bounded below by $1 - 2^{-e}$ [11]. For instance, let us fix a target recovery probability of 90%. In this case approximately $e = 4$ extra transmissions are needed, independently of the collision size. Hence, for a collision of size $k = 4$ we would need 8 transmissions, while 24 transmissions would be needed for a collision of size $k = 20$. The efficiency from the first to the second case grows from 0.5 to 0.83.

As pointed out in Section IV there may be systems in which only collisions up to a certain size $\overline{k}$ can be resolved. This can happen in systems where $M > \overline{k}$ and for heavy loads. Thus we tested one of the proposed schemes in a system with a bounded recoverable collision size and a large number of terminals. Fig. 4 shows the normalized throughput in case of slotted ALOHA with terrestrial MU PHY NC with $M = 10^5$ and different values of $\overline{k}$. A buffer with size $B = 10$ is assumed and a transmission probability $q = \frac{1}{2}$ is adopted during the RP. Note that in the system with $\overline{k} = 1$ no collision can be recovered, so it is equivalent to a slotted ALOHA. From the plot it can be seen how a relatively low value of $\overline{k} = 5$ can achieve significant gains in terms of normalized throughput with respect to a slotted ALOHA system. The decrease in throughput at high load is due to the fact that the probability of collisions of size $k > \overline{k}$ grows as λ grows. This translates into a high packet loss rate, as R is less and less capable to recover collisions as the network load grows. The value of λ beyond which the throughput starts to decrease, grows with $\overline{k}$. It is interesting to note how the corresponding breaking point is roughly equal to $\overline{k}$.

VI. Conclusions

We have proposed two new schemes for collision resolution in slotted ALOHA networks based on MU PHY NC, one for terrestrial and one for satellite networks. After describing in detail the idea of MU PHY NC and the two proposed schemes, we considered two different scenarios. In the first one the number of terminals in the network is below the maximum recoverable collision size $\overline{k}$, while in the second one the number of terminals is higher than $\overline{k}$. Simulation results showed that the proposed methods increase the system throughput with respect to slotted ALOHA in both scenarios. As future work we will carry out an analytical study of throughput and delay, and we will address issues regarding the practical implementation of these protocols.

Acknowledgments

G. Cocco is partially supported by the European Space Agency under the Networking/Partnering Initiative. D. Gündüz is partially supported by the Spanish Ministry of Science and Innovation through project JUNTOS (TEC2010-17816).

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